Rebinning and reconstruction techniques for 3D TOF-PET

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Abstract

The measured time difference in 3D Time-of-Flight (TOF) positron emission tomography (PET) makes it possible to improve the signal-to-noise ratio of reconstructed images. The improvement in signal-to-noise ratio will probably be used to reduce imaging time. To keep up with workflow there will be a need for faster reconstruction methods. A variety of reconstruction and rebinning methods have been developed in the past for 2D and 3D TOF-PET data. The TOF information makes very simple reconstruction methods possible. These allow real time reconstruction but the obtained image quality is lower. Relative fast reconstructions can be obtained using rebinning techniques. Fully 3D iterative listmode reconstruction makes no approximations but comes at the expense of long reconstruction times. Data from Monte Carlo simulations of 3D TOF-PET scanners are used to quantify differences in noise and contrast between the different methods. Real time methods are useful for direct display after or even during acquisition, but do not generate useful data for reviewing. Rebinning methods can be used to reduce the reconstruction time with a small loss in image quality and the image quality loss is quite small if good timing resolution can be achieved. Fully 3D iterative listmode reconstruction maximizes the obtained image quality and should be used if not even a small loss in image quality is acceptable. When timing resolution is improved the difference between the different methods become clearly smaller and in the limit where timing resolution is equal to spatial resolution, the methods are equivalent.

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1. Introduction

In positron emission tomography (PET) the measured data is a set of coincidences. A coincidence is measured when two photons are detected in a short time interval (typically 4–8 ns for a human PET scanner). When a photon is measured in a detector, a time window is opened during which further detection in other detectors is possible. With current commercial PET technology, the limitation on the timing resolution is about 4 ns, which constrains the positron to a 60-cm region. The accuracy on the timing measurements in current PET scanners is not accurate enough to give any information where on the line between the two detectors the annihilation took place. Therefore we assume during reconstruction that the data was acquired somewhere on the line between the two detectors. Forward and back projection is done along the lines and all points along the line have equal probability. In time-of-flight (TOF) PET, we use the coincidence window to select the coincident events, but we also try to determine the time difference between the arrivals of both photons with more precision. This can then be used to estimate the location of the position along the line where the annihilation took place. In TOF PET reconstruction we use this information. Forward and back projection is still done along the lines but all points along the line have a different probability. The maximum probability is given to the point determined by the measured TOF difference. The width of the probability distribution is determined by the expected accuracy of the time difference measurement. Mostly Gaussian distributions are assumed.

When doing reconstruction for conventional PET the noise from one voxel adds to the noise of the other voxels along the line [1]. In TOF PET the probability along the line is not uniform anymore and the noise contribution to
other voxels along the line becomes smaller. This improves the signal-to-noise ratio and increases the image quality of the reconstructions for the same number of measured counts. Different papers have investigated the improvement in image quality by the use of TOF information in PET. The major conclusion is that the gain achieved by TOF PET versus conventional PET depends on the size of the object in the FOV. The gain in NEC is proportional to the ratio of the object diameter to the full width at half maximum of the timing resolution.

2. TOF-PET systems

During the 1980s different PET systems with TOF information were developed [2–4]. The scintillators (primarily BaF$_2$) had low stopping power and low light. Therefore, these systems could not compete with non-TOF based systems based on BGO crystals. Recently there has been a renewed interest [5] in TOF-PET based on fast scintillators with a sufficiently high stopping power and high light output like LSO and LaBr$_3$ [6].

3. Mathematical description of TOF-PET kernel

Due to the imperfection of the imaging system and the backprojection process, the backprojection of a measured point will be a blurred distribution. Therefore, an inverse filter has to be applied to obtain the original distribution. In conventional PET the backprojected point spread function has a $1/r$ behaviour ($r$ the distance from the centre of the FOV).

In TOF-PET all points on the line have a different probability and the probability function is assumed to be Gaussian. Given that the timing resolution is $\Delta t$ and $c$ the speed of light, its spatial equivalent is determined by

$$\text{FWHM}_{\text{TOF}} = 2.35 \sigma = c \Delta t / 2.$$ 

If the source is at the centre of the FOV, the TOF response function for all LORs is a one-dimensional Gaussian with maximum at the source position.

$$r(t) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{r^2}{2\sigma^2} \right].$$

Therefore the point-spread function in TOF-PET is the point-spread function of ordinary PET, convoluted with this Gaussian distribution.

$$\text{psf}(r) = \frac{1}{r} \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{r^2}{2\sigma^2} \right].$$

The Gaussian distribution enters as a result of the statistical nature of the processes involved. A comparison between these two functions reveals that the mass of the TOF-PET point-spread function is more centred on its centre of gravity, giving a better-defined peak compared to the PET point-spread function. The result is that the system in TOF-PET does not smear out a point as severely as a conventional PET scanner. The signal-to-noise ratio and the image quality are thus improved.

4. Reconstruction for TOF-PET

4.1. Most likely position method

The most likely position (MLP) method is a real time method that does not take into account the angular dependence of the data. It assumes the accuracy of the time information is good enough to assign all activity directly to one point in the matrix. For an LOR with endpoints at $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ and a TOF difference $\Delta t = t_2 - t_1$, the most likely point $(x_{\text{MLP}}, y_{\text{MLP}}, z_{\text{MLP}})$ is given by

$$x_{\text{MLP}} = \frac{x_1 + x_2}{2} - \frac{c}{2} \frac{\Delta t (x_2 - x_1)}{d},$$

$$y_{\text{MLP}} = \frac{y_1 + y_2}{2} - \frac{c}{2} \frac{\Delta t (y_2 - y_1)}{d},$$

$$z_{\text{MLP}} = \frac{z_1 + z_2}{2} - \frac{c}{2} \frac{\Delta t (z_2 - z_1)}{d}.$$

(1)

where $c$ is the speed of light, and $d$ (the length of the LOR) is equal to

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$ 

(2)

The image is updated for each measured LOR with 1 at the pixel containing the most likely point calculated from the measured endpoints and the TOF difference (Eq. (1)).

4.2. Reconstruction for 2D TOF-PET

As the first TOF capable systems were built during the 80’s when 3D PET was not common, most reconstruction methods focused on 2D reconstruction. Two important methods that have been used are iterative 2D TOF MLEM and the equivalent of FBP: Confidence weighted back-projection.

4.3. Reconstruction for 3D TOF-PET

4.3.1. Hybrid methods

Different papers have also discussed hybrid methods [7–9]. These methods reduce the data to 2D data which are then reconstructed by 2D reconstruction algorithms.

The SSRB-TOF [7,9] method is basically using the same equations as the MLP method, but only uses it to determine the most likely slice $(z_{\text{MLP}})$. The time difference is projected on the 2D plane and the resulting 2D data is then reconstructed using 2D TOF-MLEM or CWBP (both modified to take the Gaussian TOF kernel into account). Here, we use SSRB-TOF followed by 2D TOF-MLEM. Recently we also proposed to use
the TOF information to mash the 2D data to a limited number of transverse angles [9].

4.3.2. Fully 3D listmode reconstruction

For each detected event the detector pair and time difference is stored sequentially in a file. Assume the number of measured events is $C$ and there are $J$ voxels in image space. For 3D listmode TOF-MLEM the update equation for the value of each voxel $j$ is

$$f_j^{(k+1)} = \frac{f_j^{(k)}}{s_j} \sum_{c=1}^{C} \sum_{j=1}^{J} \tau_{cj} a_{cj} f_j^{(k)}.$$  \hspace{1cm} (3)

The term $\tau_{cj}$ is the weight along the line of flight due to the Gaussian TOF function. It is maximal at the most likely point for that LOR. The term $a_{cj}$ is the contribution of each voxel $j$ to LOR $c$ in the absence of TOF measurement. This term is equal for 3D listmode PET. The term $s_j$ is necessary to compensate for the non-uniform sensitivity over the FOV. A similar equation is used for 2D TOF MLEM from binned data. In this case the summation in the back-projection is not over the number of measured events but over all possible TOF bins.

4.4. Reconstruction time

Simulated data were used to determine the calculation time on a single CPU (2 GHz Pentium 4). With the MLP method it is possible to process 1.4 M coinc/s, while the MLEM (one iteration) method can handle 18 k coinc/s. In the MLEM method integration along the whole line was done with the Siddon method. Faster reconstruction could be obtained by limiting the integration path to a short section of the LOR around the most likely point.

5. Influence of TOF resolution

5.1. Influence on CRC

The contrast recovery coefficient (CRC) is evaluated for the three different algorithms described: MLP, SSRR-TOF + 2D TOF-MLEM and 3D listmode TOF-MLEM. A cylinder of radius 10 and 8 cm axial length was simulated in a 3D TOF-PET scanner (60 axial rings covering 25 cm axial FOV) with perfect detectors (4 × 4 mm). In the centre of the cylinder a hot cylinder (1 cm radius and 2 cm axial length) was placed. The contrast ratio between hot source and background was 25:1. The CRC is shown in Fig. 1. For both MLEM methods the CRC was stable at iteration 8. For 3D listmode TOF-MLEM the CRC is only slightly improving when the timing resolution improves. The accuracy of rebinning improves with timing resolution and therefore the CRC of this method becomes equal to the fully 3D listmode TOF-MLEM reconstructions for timing resolutions below 300 ps. The MLP method is clearly leading to a reduction in contrast and only approaches the CRC of the other methods for timing resolutions below 100 ps. Rebinning is only leading to reduced CRC for timing resolutions above 400 ps.

5.2. Influence on convergence

A 17.5 cm radius uniform phantom containing four 1 cm diameter spheres (7 cm radius) was simulated with different timing resolution. The contrast of the 3D listmode TOF-MLEM as a function of the iteration number is shown in Fig. 2 for singles timing resolution of 100, 200, 400, 800 and 1600 ps. The points on the curve represent iterations from 1 to 32 iterations. The iteration number at which the contrast reached 4.25 (for 1600 ps this is reached at iteration 32) is plotted in Fig. 3.

The number of iterations needed increases almost linearly with the timing resolution. The same CRC is reached at lower iteration numbers when the TOF resolution becomes smaller. This results in lower noise levels in the background, which will improve detectability.
6. Conclusion

When the TOF resolution is improved the effective sensitivity is improved because for the same number of counts the same contrast is obtained with reduced noise in the background.

Good TOF information allows real time display using most likely position (MLP) method. These are however reconstructions of low contrast which are quite noisy. Preserving the listmode data allows to use 3D listmode TOF MLEM. This method gives the best contrast noise curves, but is rather slow (1 iteration of 1 million events ~50 s). It may become feasible as less iterations are needed for improved timing resolution. Furthermore speedup factors can be obtained by integrating only along a short segment of the LOR around the most likely point.

Like in current 3D (non-TOF) PET scanners the reconstruction speed can be improved by rebinning the data to 2D. The accuracy of rebinning algorithms also improves when the timing resolution becomes better. Below 300 ps the differences with 3D listmode TOF MLEM reconstruction become very small.

References